Number Systems

1.1 Introduction

Many number systems are used, such as decimal, binary, octal, hexadecimal, etc. All people are using the decimal system daily. So that, the most common used system is the decimal number system. The other number systems are used in digital systems applications. The feature which distinguishes one system from another is the number of digits which are used, and this is called the base (radix) of the system. These systems are classified according to the radix of the number system as shown below:

Base	name of number system	digits used in system
2	Binary	0, 1
8	Octal	0, 1, 2, 3, 4, 5, 6, 7
10	Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
16	Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

In general, quantities are represented as:

$N= a_{-1}\, r^{-1} + a_{-2}\, r^{-2} + \ldots + a_0\, r^{0} + a_1\, r^{1} + a_2\, r^{2} + \ldots + a_n\, r^{n}$

Where each coefficient a, can take any value of the number system digits and r is the base of the number system.

A **decimal number** system uses 10 digits to represent any quantity. The thousands, hundreds, etc., are powers of 10 implied by the position of the coefficients (symbols) in the number. The digit in the right is called Least Significant Digit (LSD), and the digit in the left is called Most Significant Digit (MSD).

….	10^4	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}	…..
….	10000	1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	….

Example: $(3752.46)_{10} = 3 \times 1000 + 7 \times 100 + 5 \times 10 + 2 \times 1 + 4 \times \frac{1}{10} + 6 \times \frac{1}{100}$

Binary Number: the decimal number can be represented in binary by arranging the 1 and 0 under weight of the binary system to get the decimal number. Each digit in binary number called a ***Bit***. The bit in the right is called Least Significant Bit (LSB), and the bit in the left is called Most Significant Bit (MSB). The positional weight of each bit is a power of 2.

….	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	…..
….	16	8	4	2	1	.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	….

Example: $(1101.11)_2 = 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 + 1 \times \frac{1}{2} + 1 \times \frac{1}{4}$

Octal Number: the decimal number can be present in Octal by arranging basic digits according to the octal system to get the decimal number where the system uses only 8 digits to represent any quantity. The positional weight of each digit is a power of 8.

….	8^4	8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}	8^{-3}	…..
….	4096	512	64	8	1	.	$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{512}$	….

Example: $(752.46)_8 = 7 \times 64 + 5 \times 8 + 2 \times 1 + 4 \times \frac{1}{8} + 6 \times \frac{1}{64}$

The **hexadecimal number system** is used commonly by designers to represent long strings of bits in the addresses, instructions, and data in digital systems. This system uses 16 digits to represent any quantity. The positional weight of each digit is a power of 16.

….	16^4	16^3	16^2	16^1	16^0	.	16^{-1}	16^{-2}	16^{-3}	…..
….	65536	4096	256	16	1	.	$\frac{1}{16}$	$\frac{1}{256}$	$\frac{1}{4096}$	….

Example: $(7EA.8F)_{16} = 7 \times 256 + 14 \times 16 + 10 \times 1 + 8 \times \frac{1}{16} + 15 \times \frac{1}{256}$

1.2 Number Base Conversion

Representations of a number in a different radix are said to be equivalent if they have the same decimal representation. It is often required to convert a number in a particular number system to any other number system, e.g., it may be required to convert a decimal number to binary or octal or hexadecimal. The reverse is also true, i.e., a binary number may be converted into decimal and so on.

1.2.1 Decimal number-to-other number systems Conversion

The conversion process of a decimal number into any number system can be done according to the following steps:

1- Separate the integer part and the fraction part.
2- Divide the integer part by the required base until the quotient of zero is obtained.
3- The column of the remainder is read from bottom to top.
4- Multiplied the fraction part with the required base until zero fraction is obtained
5- The column of integer part of result is read from top to bottom.

1.2.1.1 Decimal to binary conversion

The above steps will be applied with the base of **2.**

Example. Convert $(34.25)_{10}$ into an equivalent binary number

Solution: the integer part is 34 and can be converted as follows:

Division	**Quotient**	**Remainder**	
$34 \div 2 =$	17	0	LSB
$17 \div 2 =$	8	1	
$8 \div 2 =$	4	0	
$4 \div 2 =$	2	0	
$2 \div 2 =$	1	0	
$1 \div 2 =$	0	1	MSB

The fraction part is 0.25 and it can be converted as follows:

Multiplication	**result**	**integer part of result**	
$0.25 \times 2 =$	0.5	0	MSB
$0.5 \times 2 =$	1.0	1	LSB

Hence the converted binary number is $(100010.01)_2$.

1.2.1.2 Decimal-to-octal Conversion

Similarly, the same steps are used with the base of **8**.

Example. Convert $(35.3125)_{10}$ into an octal number.

Solution: the integer part is 35 which can be converted as follows

Division	Quotient	Remainder	
$35 \div 8 =$	4	3	LSD
$4 \div 8 =$	0	4	MSD

The fraction part is 0.3125 and it can be converted as follows:

Multiplication	Result	integer part of result	
$0.3125 \times 8 =$	2.5	2	MSD
$0.5 \times 8 =$	4.0	4	LSD

Hence the converted octal number is $(43.24)_8$.

1.2.1.3 Decimal-to-hexadecimal Conversion

The same steps are repeated with the base of **16**.

Example. Convert $(34.3)_{10}$ into a hexadecimal number.

Solution: the integer part is 34 which can be converted as follows

Division	Quotient	Remainder	
$34 \div 16 =$	2	2	LSD
$2 \div 16 =$	0	2	MSD

The fraction part is 0.3 and it can be converted as follows:

Multiplication	Result	integer part of result	
$0.3 \times 16 =$	4.8	4	MSD
$0.8 \times 16 =$	12.8	12	
$0.8 \times 16 =$	12.8	12	LSD

This is cyclic number

Hence the converted hexadecimal number is $(22.4CC)_{16}$.

1.2.2 Conversion from any number system to decimal system

The conversion process from any number system to decimal system depends on the summation of the multiplied digits by the positional weight of that system.

1.2.2.1 Binary-to-decimal Conversion

Each of the digits in the number systems discussed above has a positional weight as in the case of the decimal system in which it is a power of 2 for binary system.

Example. Convert $(10101.01)_2$ into a decimal number.

Solution.

$$N = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2}$$

$$= 1\times 16 + 0\times 8 + 1\times 4 + 0\times 2 + 1\times 1 + 0\times 0.5 + 1\times 0.25$$

$$= 21.25$$

Hence the converted decimal number is $(21.25)_{10}$.

1.2.2.2 Octal-to-decimal Conversion

The positional weight of each digit in octal number is a power of 8.

Example. Convert $(162.35)_8$ into an equivalent decimal number.

Solution.

$$N = 1\times 8^2 + 6\times 8^1 + 2\times 8^0 + 3\times 8^{-1} + 5\times 8^{-2}$$

$$= 1\times 64 + 6\times 8 + 2\times 1 + 3\times \frac{1}{8} + 5\times \frac{1}{64} = 114.453125$$

Hence the converted decimal number is $(114.453125)_{10}$.

1.2.2.3 Hexadecimal-to-decimal Conversion

The positional weight of each digit in hexadecimal number is a power of 16.

Example. Convert $(3CD.F9)_{16}$ into an equivalent decimal number.

Solution.

$$N = 3\times 16^2 + 12\times 16^1 + 13\times 16^0 + 15\times 16^{-1} + 9\times 16^{-2}$$

$$= 3\times 256 + 12\times 16 + 13\times 1 + 15\times \frac{1}{16} + 9\times \frac{1}{256}$$

$$= 973.97265625$$

Hence the converted decimal number is $(973.97265625)_{10}$.

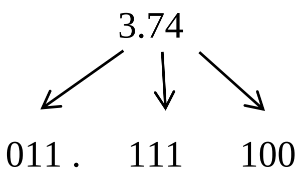
1.2.3 Conversion from Octal to Binary Number and Vice Versa

The conversion from octal to binary is performed by converting each octal digit to its three-bits binary equivalent. The eight possible digits are converted as indicated in this table.

Octal Digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

Example. Convert $(3.74)_8$ into an equivalent binary number.

Solution: by converting each digit into binary of three bits group.

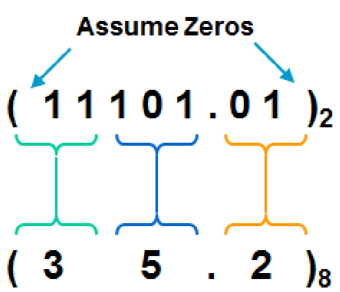


Hence the equivalent binary number is $(011.111100)_2$.

Converting from binary to octal is simply the reverse of the foregoing process. The bits of the binary number are grouped into groups of three bits starting from the LSB for integer part and starting from MSB for fraction part. Sometimes the binary number will not have even groups of three bits. For those cases, we can add one or two 0s to the left of the MSB for integer part and to the right of the LSB for fraction part.

Example: convert $(11101.01)_2$ into an equivalent octal number.

Solution:



Hence the equivalent octal number is $(35.2)_8$.

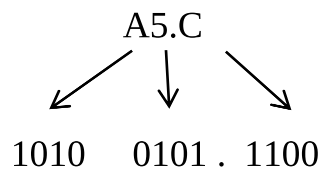
1.2.4Conversion from Hexadecimal to Binary Number and Vice Versa

The conversion from hexadecimal to binary is performed by converting each hexa digit to its four-bits binary equivalent. The sixteen possible digits are converted as indicated in this table.

Hexa	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

Example. Convert $(A5.C)_{16}$ into an equivalent binary number.

Solution: by converting each digit into binary of four bits group.

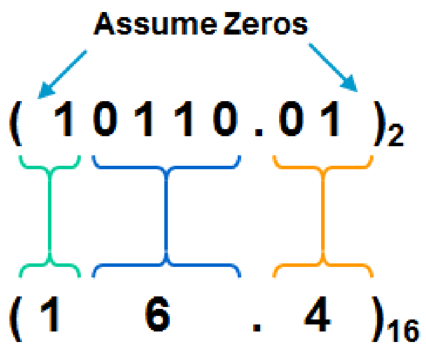


Hence, the equivalent binary number is $(10100101.1100)_2$.

Converting from binary to hexa is simply the reverse of the foregoing process. The bits of the binary number are grouped into groups of four bits starting from the LSB for integer part and starting from MSB for fraction part. Sometimes the binary number will not have even groups of four bits. For those cases, we can add one, two or three 0s to the left of the MSB for integer part and to the right of the LSB for fraction part.

Example: Convert $(10110.01)_2$ into an equivalent hexadecimal number.

Solution:



Hence the equivalent hexa number is $(16.4)_{16}$.

1.2.5 Conversion from an Octal to Hexadecimal and Vice Versa

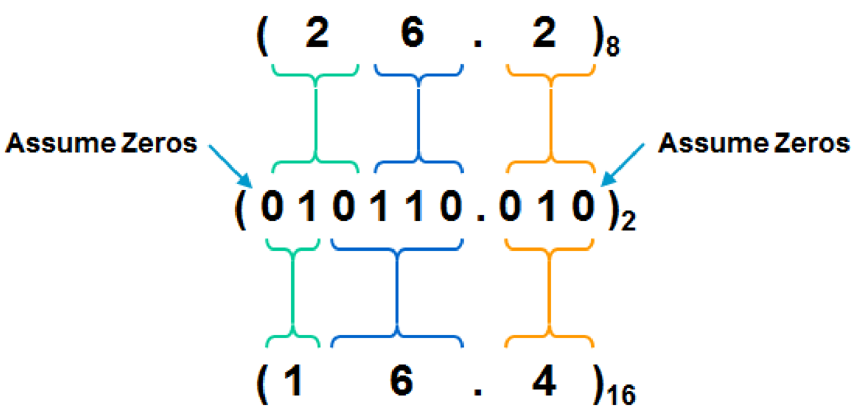
Conversion from octal to hexadecimal and vice versa is sometimes required. To convert an octal number into a hexadecimal number the following steps are to be followed:

(*i*) First convert the octal number to its binary equivalent (as already discussed above).

(*ii*) Then form groups of 4 bits, starting from the LSB.

(*iii*) Then write the equivalent hexadecimal number for each group of 4 bits.

Example: convert $(26.2)_8$ into hexadecimal.



Example: convert $(16.4)_{16}$ into octal.

Solution:

$$(\underbrace{1}\quad\underbrace{6}\ .\ \underbrace{4})_{16}$$

$$(\ \overbrace{0001}\ \overbrace{0110}\ .\overbrace{0100}\ )_2$$

$$(00\ \ \underbrace{010}\ \ \underbrace{110}\ .\ \underbrace{010}\ \ 0)_2$$

$$(\ \ \overbrace{2}\ \ \overbrace{6}\ .\overbrace{2}\ )_8$$